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A note on spatially growing three-dimensional disturbances in a free shear layer

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It does not seem to be possible to prove analytically that an incompressible, inviscid free shear layer is less unstable with respect to spatially growing three-dimensional disturbances than to two-dimensional ones. For this reason a numerical calculation for the special case of the hyperbolic tangent velocity profile was performed. It was found that even for spatially growing disturbances the amplification of three-dimensional disturbances is smaller than for two-dimensional ones.

It was shown by Squire (1933) that three-dimensional disturbances in an incompressible parallel flow are more stable than two-dimensional ones. The proof, however, was restricted to temporally growing disturbances. For inviscid free shear layers it was found by Michalke (1965) and Freymuth (1966) that the agreement between the results of stability theory and experiment is much better, if spatially growing disturbances were assumed in the theory. However, for this case, the Squire theorem is not applicable. Therefore the question whether three-dimensional disturbances are more stable or not cannot generally be answered. In the following, this problem will be treated for the special case of the hyperbolic tangent velocity profile.

For a three-dimensional disturbance the velocity component v normal to the basic parallel flow $U(y)$ has the form

$$v(x,y,z,t) = i\alpha\phi(y)\, e^{i(\alpha x+\gamma z-\beta t)}. \tag{1}$$

The amplitude function $\phi(y)$ has to satisfy the inviscid disturbance equation

$$\phi'' - \left[\frac{U''}{U-\beta/\alpha} + \alpha^2 + \gamma^2\right]\phi = 0. \tag{2}$$

Considering a free shear layer the boundary conditions are given by

$$\phi(\infty) = \phi(-\infty) = 0. \tag{3}$$

For physical reasons γ has to be real and is the spanwise wave-number of the disturbance.

Assuming temporally growing disturbances, α has to be real and is the wave-number in the basic flow direction x, while $\beta = \beta_r + i\beta_i$ is complex. β_r is the disturbance frequency and β_i the temporal growth rate. The eigenvalue problem posed by (2) and (3) then yields the complex eigenvalues β by a relation

$$\beta/\alpha = f(\sqrt{(\alpha^2+\gamma^2)}), \tag{4}$$

where the complex function f for a fixed basic velocity profile $U(y)$ is universal, whether γ is zero or not. Therefore the maximum of the temporal growth rate β_i for $\gamma \neq 0$ cannot exceed the value for $\gamma = 0$.

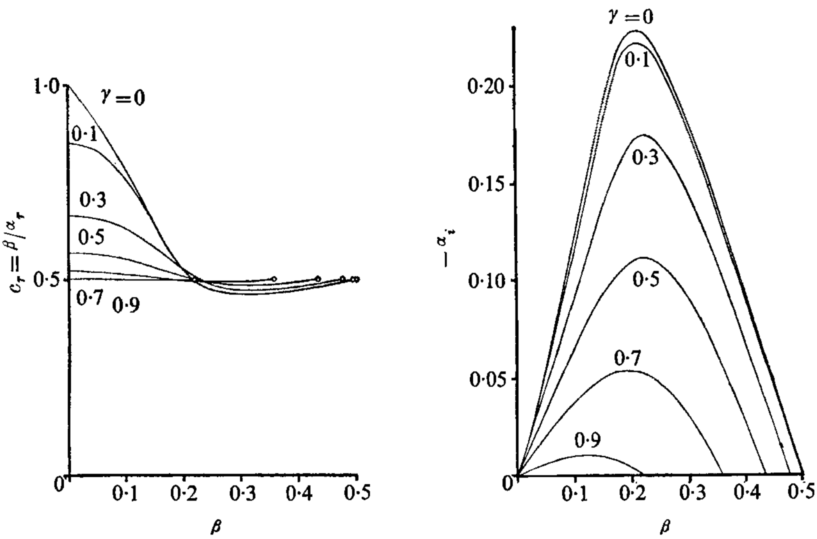

FIGURE 1. Phase velocity and spatial growth rate of the tanh velocity profile for various spanwise wave-numbers γ.

Contrary to this, for spatially growing disturbances β has to be real and is the disturbance frequency, while $\alpha = \alpha_r + i\alpha_i$ is complex. α_r is the wave-number in basic flow direction x and $-\alpha_i$ is the spatial growth rate. Now the complex eigenvalues α are given by a relation

$$\alpha = g(\beta, \gamma), \tag{5}$$

in which the influence of γ is not as simple to discuss as that found in (4) for temporally growing disturbances.

The complex eigenvalues α were computed for the basic velocity profile

$$U(y) = 0{\cdot}5[1 + \tanh y], \tag{6}$$

using a method similar to that of Michalke (1965). The disturbance phase velocity $c_r = \beta/\alpha_r$ and the spatial growth rate $-\alpha_i$ as a function of the frequency β for various spanwise wave-numbers γ are shown in figure 1. One can see that the spatial growth rate for fixed frequency is smaller for $\gamma \neq 0$ than for $\gamma = 0$. Therefore the three-dimensional disturbances are less unstable than two-dimensional disturbances. It is well known that for the neutral case we have $\alpha_r^2 + \gamma^2 = 1$ and $\beta/\alpha_r = 0{\cdot}5$, which yields the neutral frequency $\beta = 0{\cdot}5\sqrt{(1-\gamma^2)}$. On the other hand, for $\beta \to 0$, $\beta/\alpha = c$ remains finite and unequal to zero. Therefore (2) reduces to

$$\phi'' - \left[\frac{U''}{U-c} + \gamma^2\right]\phi = 0. \tag{7}$$

The eigenvalues of (7) and (3) are then equivalent to those for two-dimensional temporally growing disturbances, if α is replaced by γ. Therefore we find for the profile (6) that

$$c = 0{\cdot}5 + ic_i(\gamma), \tag{8}$$

where $c_i(\gamma)$ was calculated by Michalke (1964). Hence the phase velocity $c_r = \beta/\alpha_r$ of three-dimensional disturbances for $\beta \to 0$ is given by

$$c_r = 2[\tfrac{1}{4} + c_i^2(\gamma)] \tag{9}$$

and the spatial growth rate by

$$-\alpha_i = \frac{c_i(\gamma)}{\frac{1}{4} + c_i^2(\gamma)}\beta. \tag{10}$$

It is assumed that these results obtained for the tanh profile are also significant for other profiles, and it is supposed that for any incompressible free shear layer the amplification of spatially growing, three-dimensional disturbances is smaller than that of two-dimensional disturbances. This seems to be confirmed by Gropengießer (1969), who treated the instability of a compressible free shear layer and found the same result, if the Mach number tends to zero.

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